...miter using this principle can be con-...d at any frequency, depending only ...e availability of an adequate dc mag-...eld, whose amplitude is somewhat ...er than the ferrimagnetic resonance ... Theoretical considerations show that ...lling factor, which is proportional to the ...ple volume divided by the cavity vol-..., must be large in order to achieve limit-...over a significant range of power. The ...lability of large single crystals of yt-...m iron garnet now makes it possible to ...ain these large filling factors along with ...ow linewidths.

Fig. 1 is a schematic of a 4 kMc model ...this limiter which used a polished sphere ...single crystal YIG, 0.260 inches in di-...eter. The desired limiting value was 14 ...m. This level can be adjusted to as low ...3 dbm with the dc magnetic field. A ...duced-height (0.400 in) transmission cav-...with a loaded Q of 200 was operated in ...TE_{101} mode with the sample placed in ...maximum RF magnetic field close to the ...e wall of the cavity. At low power levels ...e sample has very little effect on cavity ...nsmission properties. Above a well-de-...ed threshold level sample losses increase ...pidly and limit the transmission power ...vel.

Temperature changes can have large ...fects on the characteristics of this device. ...he limiting power level changes with $4\pi M$, ΔH_k, H_{dc} and H_a, the anisotropy field.[7] In ...ddition, there are low-level insertion loss ...ariations due to thermal cavity detuning ...fects. These include cavity expansion and ...RF susceptibility variations dependent on $4\pi M$, ΔH_k, H_{dc}, and H_a. A major source of ...thermal changes in limiter performance can ...e avoided by the use of a spherical sample, ...or which the ferromagnetic resonance fre-...quency is independent of the saturation ...magnetization and its associated tempera-...ure variation. It can also be shown that ...rientation of the anisotropy field can be ...sed to minimize further the temperature ...ensitivity. The samples used here were of ...purity such that ΔH_k had only a slightly ...egative temperature coefficient at room ...emperature. For these samples orientation ...f the magnetically hard [100] crystallo-...graphic axis along the biasing field gave ...optimum temperature characteristics.

The limiter performance over the design ...temperature range from 55° to 120°F is ...shown in Fig. 2. The sharp break in char-...acteristic at the threshold is important for ...efficient power leveling. It is dependent on ...the use of single-crystal narrow-linewidth ...material and the surface polish. In this case ΔH_k was approximately 0.15 oe at 4 kMc. ...Spike leakage was observed but not in-...vestigated; however, this limiter should have ...haracteristics similar to the coincidence ...imiter[2] in this respect.

Because of weight limitations in the ...satellite application, the cavity was fabri-...ated from silver-plated magnesium result-...ing in a total weight, including magnet, of ...ounces. While the limiter was designed at ...particular frequency with particular re-...quirements, it should prove useful in other applications at different frequencies requiring precise control of power level.

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R. L. Comstock
W. A. Dean
L. J. Varnerin
Bell Telephone Labs.
Murray Hill, N. J.

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Power-Aperture and the Laser*

The author has heard frequent reference to the power-aperture product criterion for radar surveillance, but has not yet seen in print a reasonably general proof of the relation, as simple as it is. The situation is particularly important at present with the development of the laser, its very narrow beamwidth and the consequent possibility of a very long range optical radar with a small aperture. It is necessary, therefore, to consider the limitations of this device, particularly for surveillance purposes. To anticipate the result: the surveillance of a given number of square degrees of coverage per second out to some range with a given input noise, requires at least the same average power-aperture product at optical as at microwave frequencies.

Large Spot Size

We consider first the usual radar case in which the beamwidth is much larger than the target (large spot size). $E_T = P_T\tau$ is the transmitted energy in an element of solid angular resolution $\Delta\Omega$ during a total coherent time of integration τ; the total coherent time τ may be broken up and distributed over a longer time interval provided the returns from an individual target can be processed coherently. P_T is the average transmitted power during time τ and corresponds to the peak pulse power if the power is emitted in uniform rectangular pulses. For a CW fence radar, τ would ideally correspond to the time of passage of the target through the fence. The transmitting antenna gain $G_T = 4\pi/\Delta\Omega$.

The basic radar equation, in terms of the returned energy, E_R, in time τ, is

$$E_R = \frac{E_T G_T \sigma A_R}{(4\pi R^2)^2} = \frac{E_T \sigma A_R 4\pi}{(4\pi)^2 R^4 \Delta\Omega},$$

where A_R = receiving antenna aperture area.

At the output of a matched filter, the signal-to-noise energy or power ratio is $2E_R/N$, where N is the input noise power per cps of bandwidth (noise spectral density). Therefore, if we denote by C the signal-to-noise power margin required for some preassigned probability of detection,

$$C = \frac{2E_R}{N} = \frac{2E_T\sigma A_R}{4\pi N R^4 \Delta\Omega}.$$

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If n is the total number of elements of angular resolution of the transmitting antenna swept out during a surveillance scan interval t and if E_G is the total energy radiated during t, then $nE_T = E_G$, and

$$C = \frac{2E_G\sigma A_R}{4\pi N R^4 n\Delta\Omega} = \frac{2E_G\sigma A_R}{4\pi N R^4 \Omega}; \quad n\Delta\Omega = \Omega.$$

$E_G = P_A t$, where P_A is the average transmitted power in a scan interval, so

$$C = \frac{2P_A t\sigma A_R}{4\pi N R^4 \Omega} \text{ or } 2\pi N C \frac{R^4}{\sigma}\left(\frac{\Omega}{t}\right) = P_A A_R.$$

This is the surveillance equation for large spot size, and shows that the solid angular coverage swept out per second is proportional to the product of the average transmitted power and the receiving aperture.

There are, of course, other limitations not expressed in the previous equation, such as mechanical limitations on speed of solid angular coverage, and assurance that the receiving aperture is in the correct angular position to receive the return echo at the time of arrival.

In an actual radar there will be losses, which can be lumped in with C. An important corollary is obvious from the above theorem, by virtue of the fact that $(\Sigma P_i)\cdot(\Sigma A_i) > \Sigma P_i A_i$, for $i > 1$. For a given rate of coverage out to a given range for the case of separate radars, not synchronized or coherent with one another, the smaller the number of radars, the smaller the total power-aperture requirements; in fact, if line-of-sight requirements permit, one radar is the most efficient. For example, if we took an extreme case of 100 radars, then the system would require ten times as much total aperture area and ten times as much total average power as would the single radar installation.

With a laser radar, in the case of large spot size, the same fundamental coverage considerations apply as for a microwave radar. Therefore, unless the assumptions of the foregoing analysis can be controverted for the laser, since we will need as high power and as large aperture for the optical as for microwave radar, there is little advantage to the laser for long range surveillance, and we will probably continue into the indefinite future with radars in the general microwave region for this purpose. Of course, in tracking with large spot size, it is true that a laser radar with a small aperture can track to the same range as a microwave radar with equal power and noise and much larger aperture.

Small Spot Size

It has been pointed out that the laser is capable of such narrow beamwidth that the spot size can be smaller than the target, so that all the power in the transmitted beam is intercepted by the target, resulting in an inverse-square optical radar range relation as compared to the inverse-fourth for the microwave radar. This inverse-square relation may be true for relatively short ranges, but at a long range such as 1000 nautical miles, for an optical-wavelength laser with a 1-cm aperture, the spot size due to the diffraction-limited beamwidth is about 300 feet across. A large percentage of space objects will be considerably less than this. Thus, for space applications over appreciable ranges, for the most part, with the small

apertures presently discussed for laser radars, the spot sizes will be large, and the conventional inverse-fourth power radar equation holds. Laser radar apertures will have to be raised to linear dimensions of the order of meters rather than centimeters to derive what advantages there are from small spot size. However, once we get to the larger dimensions cited, one of the main advantages of the optical radar, small size, is lost. However, for shorter ranges, such as 10 nautical miles, a tremendous power or aperture advantage is theoretically available over a microwave radar subject to the inverse-fourth power restriction.

It is important, though, to realize that in the whole preceding discussion we have given minimum attention to target cross section as a function of wavelength and spot size, and the effect of motion and consequent fluctuations. The whole subject is too involved to be treated sketchily as it must in a contribution such as the present one. However, one simple example will be given, to bring out the kind of problem that exists with small spot size.

Suppose that one had a 100-foot perfectly-reflecting smooth sphere. The only region of the sphere which would reflect back to the source would be a small area surrounding the radius vector from the source to the center of the sphere. Any other region of the sphere would be at such an angle to the radius vector that it would reflect signal away from the source. With large spot size, because of beamwidth much greater than the target, as long as the sphere is within the beam there will always be a radius vector within the beam perpendicular to some part of the sphere, so that some portion of the radiated energy is bound to be reflected back. However, with small spot size, unless the spot is in the region of the sphere perpendicular to the radius vector, there will be essentially no reflection back to the source; the energy will be specularly reflected in some other direction. The return signal strength is tremendous when the beam is exactly centered on the sphere, but goes essentially to zero when off-center.

M. D. RUBIN
The MITRE Corp.
Bedford, Mass.

Parametric Amplification and Oscillation at Optical Frequencies*

Recent experimental work[1] using intense optical fields produced by a maser has indicated that it is possible to obtain variable parameter interaction in a solid. It follows that the processes of amplification and oscillation utilized in microwave devices may be extended to the optical frequency range. Specifically, we propose that coherent optical energy may be generated at subfrequencies if a nonlinear dielectric material is driven by an optical maser "pump," such as a ruby. Here we derive the conditions for oscillation in a simple resonant system based on the observed experimental data for second harmonic generation.[1-3]

Consider a resonant structure composed of two parallel, highly reflecting surfaces bounding a medium of nonlinear dielectric material, such as quartz. We shall excite the medium with a traveling-plane wave of frequency f_p and by parametric excitation produce standing waves in the medium at frequencies f_s and f_i subject to the condition that $f_p=f_s+f_i$. Here it is assumed that the thickness of the medium l is such that there exist resonant modes at f_s and f_i. It is also assumed that the reflectivity of the walls is small at the pump frequency so that the pump wave may propagate through the structure without appreciable reflection. Under these conditions, it may be shown[4] that the rate of change of amplitude of the signal wave E_s due to interaction of the "idler" wave E_i with the pump is given by

$$\frac{\partial E_s}{\partial t} = \frac{1}{4\epsilon l}\int_0^l e^{-jk_s z}\left(\frac{\partial P_s}{\partial t}\right)dz$$
$$= \frac{j\omega_s\gamma_{si}E_i^*E_p}{4\epsilon l}\int_0^l e^{(j(k_p-k_s-k_i)z}dz \quad (1)$$

where k_s, k_i, and k_p are the respective wave vectors $(2\pi/\lambda)$ for the signal, idler, and pump; and it has been assumed that the polarization of the nonlinear medium at frequency f_s is

$$|P_s| = \gamma_{si}|E_i||E_p| \quad (2)$$

where γ_{si} is a function of the three frequencies. Taking into account the Q of the cavity at the frequency f_s we obtain the equation,

$$\frac{\partial E_s}{\partial t} = \alpha_{si}E_i^* - \frac{\omega_s}{2Q_s}E_s, \quad (3)$$

and a similar equation for the idler wave,

$$\frac{\partial E_i}{\partial t} = \alpha_{is}E_s^* - \frac{\omega_i}{2Q_i}E_i \quad (4)$$

where

$$\alpha_{si} = \frac{j\omega_s\gamma_{si}E_p}{4\epsilon l}I_{si}(l) \quad (5)$$

and $I_{si}(l)$ is the "coherence" integral of (1). Now for oscillation, the rate of growth of the signal and idler waves should be zero or greater, and setting (3) and (4) equal to zero yields

$$\alpha_{is}\alpha_{si}^* - \frac{\omega_i\omega_s}{4Q_sQ_i} = 0. \quad (6)$$

Using the Q of a planar cavity with power reflectivity R given by

$$Q_s = \frac{k_s l}{1-R}; \quad (7)$$

and setting $\omega_s \cong \omega_i \cong \omega_p/2$, we obtain

$$\frac{\omega_s^2\gamma_{si}^2E_p^2I_{si}^2(l)}{4\epsilon^2l^2} > \frac{\omega_s^2(1-R)^2}{k_s^2l^2} \quad (8)$$

or

$$\gamma_{si}^2E_p^2I_{si}^2(l) > \frac{4\epsilon^2\omega_s^2(1-R)^2}{k_s^2} \quad (9)$$

as the condition for oscillation at the frequencies f_s and f_i.

In a similar manner we may calculate the second harmonic electric field for traveling wave of frequency f_p obtaining

$$E_{2p} = j\omega_p\sqrt{\frac{\mu}{\epsilon}}\gamma_{2p}E_p^2\int_0^l e^{j(2k_p-k_{2p})z}dz \quad (10)$$

where the polarization at frequency $2f_p$ given by

$$|P_{2p}| = \gamma_{2p}|E_p|^2, \quad (11)$$

and the generation takes place over a path length l. We now define the efficiency of second harmonic power generation as

$$\eta = |E_{2p}|^2/|E_p|^2 = \omega_p^2\frac{\mu}{\epsilon}\gamma_{2p}^2E_p^2I_{2p}^2(l) \quad (12)$$

with $I_{2p}(l)$, the "coherence" integral of (10). For a practical experiment, with the proper choice of materials, the values of and the coherence integral $I(l)$ should be approximately the same for second harmonic generations as for parametric mixing. Thus, the inequality of (9) reduces to

$$\eta > (1-R)^2 \quad (13)$$

for the same length, l, and pump amplitude E_p. Recent experiments[1-3] indicate that η can be of the order of 10^{-6} indicating that the reflectivity of the cavity walls should be 99.9 per cent or higher for oscillation. This reflectivity should be obtainable using multiple dielectric layer films; higher pump fields using advanced techniques should relax the above requirements.

We have here considered a special case of subfrequency generation using a simple cavity geometry and a traveling pump wave. There are many other possible configurations for such cavities utilizing a standing wave pump, for example, or taking advantage of the tensor properties of the crystal to obtain longer interaction lengths, as described by Giordmaine[2] and Maker.[3] ... felt that the possibility of coherent generation of lower frequencies as shown by calculation offers great promise as an alternative source of long wavelength energy at frequencies where direct maser action is not feasible. In addition, upon the availability of continuous high-power sources, amplifiers may also be built using the above techniques. Experiments are underway to verify the above predictions.

R. H. KINGSTON
Lincoln Laboratory†
Mass. Inst. Tech.
Lexington, Mass.

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